Temperature Dependence of the Conductivity and Minimum Quantum Mobility of a Highly Disordered Dilute Two-Dimensional Electron Gas

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We report the first observations of magnetic-field-independent corrections to the conductivity in gas atom scattering of electrons on solid H_2 These incoherent corrections, which are substantially larger than the weak-localization terms at high impurity densities, are estimated via an electron-trapping model. The model has essentially no adjustable parameters and gives a self-consistent extrapolation between the weak- and strong-disorder limits. In the regime in which trapping is dominant we observe a residual electron mobility $\mu_{\rm res} \sim T^{-1}$ associated with a lower quantum limit of diffusion.

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The nearly ideal two-dimensional electron gas (2DEG) formed by depositing electrons on solid hydrogen at liquid-helium temperatures has proven to be a unique and extraordinarily rich system in which to study localization in a classical 2DEG.[1-6] Previous magnetotransport studies of electrons on hydrogen crystals, which were purposely disordered by either photoinduced surface defects[1,7] or ambient helium-gas atoms,[2] have demonstrated that both weak- and strong-localization effects are readily observed in the system. Nominally, weak localization was identified with the appearance of a perturbatively small negative magnetoresistance at magnetic fields less than 1 T. Strong-localization effects were characterized by the appearance of a disorder-dependent energy threshold for conduction which, with sufficiently strong disorder, can be several times larger than k_BT. Notwithstanding an extensive study[1,2,8] of the above limiting characterizations of the effects of disorder on the transport properties of electrons on hydrogen, a consistent extrapolation between the weak- and strong-localization regimes as it related to the measured conductivity and mobility had remained elusive and it became evident that temperature-dependence studies were needed.

Gas atom scattering is mediated by a well-known and tunable random potential for which we have an independent theoretical estimate of the strength of the disorder. In this respect the system provides a powerful and straightforward experimental probe of the quantum corrections to the Boltzmann conductivity as function of the impurity density. Recently, Kirkpatrick and Belitz[9] have shown via an exact perturbation calculation that in a 3D disordered, noninteracting electron gas the leading-order correction to the dc conductivity is proportional to $\hbar/E_F\tau_0$ and *not* to the usual $(\hbar/E_F\tau_0)^2$ correction of weak-localization theory,[10] where E_F is the Fermi energy and τ_0 is the elastic-scattering time. Though the nature of the 2D static impurity corrections are not yet known,[11] the $\hbar/E_F\tau_0$ term is clearly evident in the 3D electron-gas atom scattering data of Schwarz[12] and we believe that an analogous term is evident in the 2D data described below. These incoherent impurity scattering terms do not necessarily require the existence of time-reversal symmetry and therefore cannot be fully probed by the application of a magnetic field. Thus to investigate the dc impurity corrections one must be able to sweep the impurity density which is easily done in gas atom scattering.

The details of our technique for measuring the conductivity and mobility of the 2DEG have been published elsewhere.[1,7] Magnetoconductance measurements were made as a function of ambient helium-gas density, n_g, at several different temperatures. All of the hydrogen crystals used in this study had an intrinsic $T=4$ K mobility greater than 0.5 $m^2/V\,s$ and at gas densities greater than approximately $n_g=10^{20}$ cm^{-3} the scattering was predominantly from gas density fluctuations. The measured conductivity was fitted by the following modified Kubo formula:[4]

$$\sigma=\frac{e^2n_0}{m_{\rm el}(k_BT)^2}\int_{E_c}^{\infty}\frac{e^{-E_c/k_BT}}{1+(\mu B)^2}\tau_0(E-E_c)dE\,, \qquad (1)$$

where B is the applied magnetic field, e is the electron charge, n_0 is the 2D electron number density, $m_{\rm el}$ is the bare electron mass, $\mu=e\tau_0/m_{\rm el}$ is the electron mobility, and E_c is a localization cutoff which accounts for both strong- and weak-localization effects. All of the disorder-dependent physics, at least in the low-carrier-density limit, is carried in E_c. We have chosen a phenomenological form for E_c which gives a consistent extrapolation between the perturbative and highly disordered regimes,[1,4,8]

$$E_c(B,n_g)=E_c^0(n_g)+\frac{e\hbar}{2\pi m_{\rm el}\mu}\frac{1}{k_BT}\int_0^{\infty}\left[\psi\left(\frac{1}{2}+\frac{\hbar m_{\rm el}}{4eBE\tau_0^2}\right)-\psi\left(\frac{1}{2}+\frac{\hbar m_{\rm el}}{4eBE\tau_0\tau_\phi}\right)\right]dE\,, \qquad (2)$$

where τ_ϕ is the electron dephasing time and ψ is the digamma function. The second term on the right-hand side of Eq. (2) is that of the usual coherent backscattering formalism where one observes a perturbative, field-dependent negative contribution to the conductivity in the presence of disorder. In zero field the thermal average over the digamma functions becomes $\ln(\tau_\phi/\tau_0)$ and in high fields becomes zero. In Ref. 2 we used only this term for E_c with Saitoh's[13] semiclassical gas atom scattering expression for μ,

$$\mu_{\rm gas}^{-1} = 3\pi^2\hbar a^2 n_g/2e\langle z\rangle\,, \tag{3}$$

to account for the zero-field conductivity as a function of gas density where $a = 0.06$ nm is the electron-helium scattering length and $\langle z\rangle = 1.7$ nm is the effective Bohr radius of an electron above the hydrogen surface. Though we obtained quite satisfactory fits to $\sigma(n_g)$ with $E_c \sim n_g$, the values of μ used in this procedure are inconsistent with the detailed magnetoresistance data described below where μ^{-1} is seen to vary sublinearly with n_g at high gas densities. A more serious concern with this analysis lies in the high-field behavior of the second term. If it alone is used for E_c then Eq. (2) predicts $E_c \simeq 0$ at high fields independent of the strength of the disorder and one is led to the physically unrealistic conclusion that there can be no localized states in the high-field limit.[14] Thus, we have included an additional localization threshold in Eq. (2), E_c^0, which accounts for electrons that are completely localized in deep potential fluctuations.

The interaction between an electron and the gas is well represented by the optical potential,[15] $\Delta V_0 = (\hbar^2/m_{\rm el})\times 2\pi a \Delta n_g$. The binding energy of an electron in a 2D quantum well of radius L_0 and depth ΔV_0 is $E_B \approx \Delta V_0(L_0) - \hbar^2/2m_{\rm el}L_0^2$. For our case the helium gas behaves as a nearly ideal gas and the scale dependence of ΔV_0 is given by $\langle \Delta n_g^2\rangle \approx n_g/\langle z\rangle \pi L_0^2$, where the denominator is the approximate volume of the electron wave function. A reasonable estimate of E_c^0 is obtained by maximizing E_B with respect to L_0,

$$E_c^0 \simeq (E_B)_{\max} \simeq \frac{\hbar^2}{m_{\rm el}} \frac{2\pi a^2}{\langle z\rangle} n_g = \gamma n_g\,, \tag{4}$$

where $\gamma/k_B \approx (1.3\ {\rm K})/(10^{20}\ {\rm cm}^{-3})$. Note that Eq. (4) predicts the same linear dependence on n_g as does the weak-localization correction in Eq. (2) with Eq. (3) substituted in for μ. Equation (4), however, has no field dependence and its derivation does not depend upon the existence of extended states nor upon $\mu^{-1} \sim n_g$ at all gas densities. A similar analysis for a 3D quantum well predicts $E_c^{0\,3D} \sim n_g^2$, in agreement with the high-density behavior of the 3D data of Schwarz.[2,12]

An important assumption will be made in interpreting the data with the above model. We will approximate the average electron kinetic energy of that portion of the Boltzmann distribution with $E > E_c$ as being k_BT, independent of the size of E_c. For a detailed qualitative justification of this rather subtle restriction on the dynamics of the system, see Ref. 1. Assuming μ to be a relatively weak function of energy (μ is independent of energy in the weak-disorder limit) and integrating Eq. (1) we have

$$\sigma = \frac{e\mu n_0 \exp[-E_c(B,n_g)/k_BT]}{1+(\mu B)^2}\,. \tag{5}$$

Fits were made to magnetoresistance data taken at various temperatures and helium densities using Eq. (5) in which μ, E_c^0, and τ_ϕ/τ_0 were independently varied for the best fit. For the most part, μ was determined by the high-field slope of the magnetoresistance, τ_ϕ/τ_0 by the size of the low-field negative magnetoresistance, and E_c^0 by the overall scale of σ_0/σ, where σ_0 is the conductivity at $n_g = 0$. Note that in the low-density limit, $E_c \ll k_BT$ in Eq. (5) and one recovers the usual field-dependent weak-localization correction to the conductivity.

Shown in Fig. 1 are two magnetoresistance plots at approximately the same gas density, $n_g = 3.6\times 10^{20}$ cm^{-3},

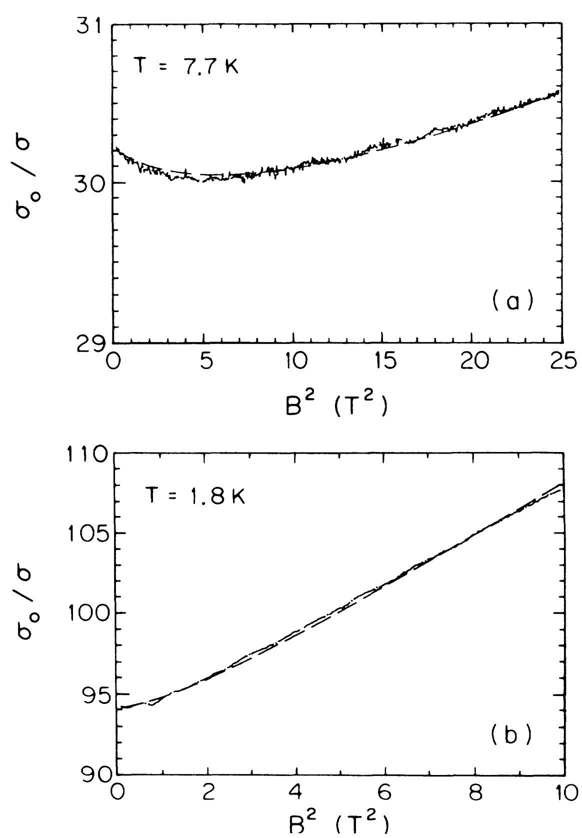

FIG. 1. Magnetoresistance of the 2DEG normalized by its $n_g = 0$ value. Both data sets are at $n_g = 3.6\times 10^{20}$ cm^{-3}. The dashed lines are the best fits to the data using Eqs. (5) and (2). The fitting parameters are $\mu = 0.056$ m^2/V s, $\tau_\phi/\tau_0 = 6.22$, $E_c^0/k_B = 4.6$ K for the $T = 7.7$ K data; $\mu = 0.16$ m^2/V s, $\tau_\phi/\tau_0 = 5.1$, $E_c^0/k_B = 4.9$ K for the $T = 1.8$ K data.

but at different temperatures. The dashed lines are the predictions of Eq. (5) with μ, τ_ϕ/τ_0, and E_c^0 varied for the best fit. The fits not only give roughly the same value of E_c^0 at each temperature but the value is in good agreement with Eq. (4). Though the upper plot represents a significantly less disordered 2DEG by virtue of its higher temperature, the mobility is smaller than the lower-temperature data. This peculiar behavior was seen in all of the data at gas densities in which $E_c \gtrsim k_B T$. We also reported similar behavior for electrons on bare H_2 in which there appeared to be a limiting mobility which varied inversely as the temperature.[1] However, there is always some doubt as to the homogeneity of light-induced surface disorder and if some portions of the crystals reported in Refs. 1 and 7 remained relatively smooth then the reported limiting mobility may have appeared as an artifact. On the other hand, if the residual mobility was indeed real, it should also be seen in gas atom scattering data where one is absolutely assured of homogeneous disorder.

To this end we have measured the high-field mobility as a function of helium-gas density at $T=1.8$, 4.0, 5.1, and 7.7 K. In Fig. 2, we have plotted the change in inverse mobility due to helium gas $\Delta\mu^{-1}=\mu^{-1}-\mu_{\text{surface}}^{-1}$, as a function of gas density along with the semiclassical inverse mobility as given by Eq. (3) (the $T=4$ K second virial coefficient is included in the theoretical curve). Note that at each temperature the mobility is somewhat lower than predicted at low gas densities but eventually crosses the classical curve at high n_g. Indeed, for the ^{3}He-gas atom scattering data taken at 1.8 K the mobility is approximately 2.5 times higher than expected classically at $n_g \simeq 5\times10^{20}$ cm^{-3}. The conductivity, however, is about a factor of 15 *lower* than expected classically. The data also clearly show the inverse temperature dependence of the saturated mobility as first suggested in Ref. 1. We believe that this saturation behavior represents a quantum effect. The magnetic field only probes that portion of the distribution with $E > E_c$ and in strong disorder (i.e., $E_c \geq k_B T$) those electrons above E_c are near the quantum limit of diffusion associated with electrons scattering on length scales of order of their de Broglie wavelength. Since the average kinetic energy of the conduction electrons is always $k_B T$, this minimum temperature-dependent length scale is preserved even in strong disorder. Using the Einstein relation, the minimum mobility before localization should be of order[1]

$$\mu_{\text{res}} \simeq e\hbar/2m_{\text{el}}k_B T \qquad (6)$$

given a minimum 2D electron diffusivity of $D \simeq \hbar/2m_{\text{el}}$. Until our most recent measurements there was still some question as to whether Eq. (6) represented a residual quantum mobility or just the mobility at which strong-localization effects begin to become important. The latter point of view was taken in Refs. 2 and 8 primarily as a justification for modeling the data with only the second term in Eq. (2). The data in Fig. 2, however, provide unequivocal evidence that μ_{res} does in fact represent a true limiting mobility. We have plotted the saturation values of $\Delta\mu^{-1}$ as a function of T in Fig. 3 and they are indeed linear with a slope that corresponds to $D \simeq \frac{1}{3}\hbar/m_{\text{el}}$ in very good agreement[16] with Eq. (6).

The existence of this residual mobility has important experimental ramifications. For instance, it masks the negative magnetoresistance in Fig. 1(b), and it seems likely that low-temperature backscattering studies will have to be performed in a Hall geometry in order to remove the $(\mu B)^2$ term in the denominator of Eq. (1). Our results also suggest that transport measurements in extremely low-carrier-density[17] GaAs 2DEG's and other nondegenerate systems[12] will show similar effects of the mobility being dominated by the conducting part of the

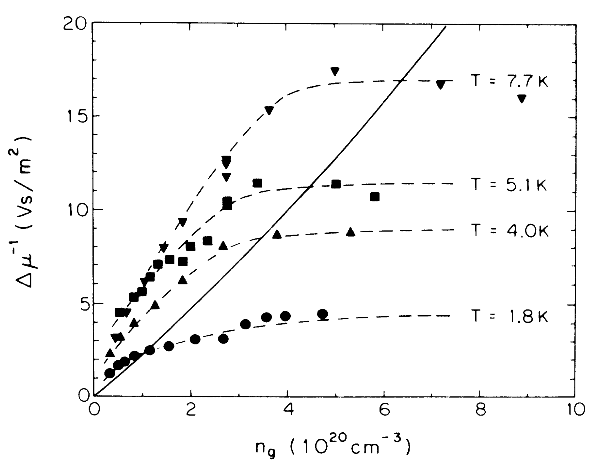

FIG. 2. The gas atom scattering contribution to the inverse mobility as a function of helium-gas density at several different temperatures. The dashed lines are provided as a guide to the eye. The solid line is μ_{gas}^{-1} given in Eq. (3). ^{3}He gas was used in the $T=1.8$ K data.

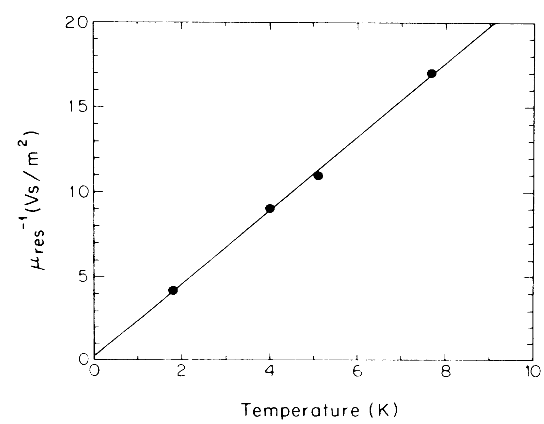

FIG. 3. The inverse residual mobility as a function of temperature. The solid line is a best linear fit to the data. The slope is interpreted as k_B/eD and gives $D \simeq \frac{1}{3}\hbar/m_{\text{el}}$.

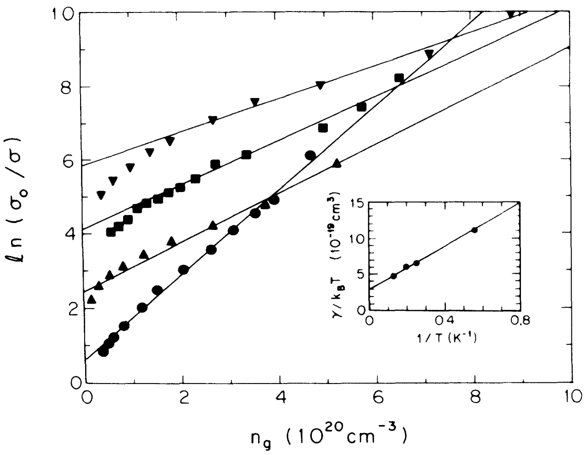

FIG. 4. The conductivity normalized by its $n_g=0$ value as a function of gas density. The curves have been offset to clearly show the temperature dependence of the slopes (●, $T=1.8$ K; ▲, $T=4$ K; ■, $T=5.1$ K; ▼, $T=7.7$ K). The solid lines are linear best fits to the data points for which $n_g > 2\times 10^{20}$ cm^{-3}. Inset: Slopes as a function of inverse temperature where γ is defined in Eq. (4).

distribution if the disorder is sufficiently strong.

The temperature dependence of the conductivity was also studied and compared to Eq. (5) which predicts activated-like behavior. Though surface scattering from defects on the hydrogen and adsorbed helium[18] is significant at small gas densities, the density dependence of E_c can be extracted from the slopes of $\ln(\sigma_0/\sigma)$ vs n_g at high gas densities. Shown as solid lines in Fig. 4 are linear best fits to the data for $n_g > 2\times 10^{20}$ cm^{-3}. Because of surface scattering effects the intercepts in these data are not meaningful and the curves have been shifted to clearly show the temperature dependence of the slopes. In the inset of Fig. 4 we have plotted the slopes as a function of T^{-1}. The slopes are linear in T^{-1} and give an activation temperature $T_c=1.5$ K at $n_g=10^{20}$ cm^{-3}. This should be compared with Eq. (4) which predicts $T_c^0 \sim 1.3$ K at $n_g=10^{20}$ cm^{-3}. The nonzero intercept in the inset is interesting and probably reflects the apparent linear T dependence of μ^{-1} in Eq. (2) at gas densities well below mobility saturation; see Fig. 2. The necessity of introducing the parameter E_c^0 into the model is especially evident in the 1.8-K data where the second term in Eq. (2) (i.e., the weak-localization term) is never greater than 1.5 K due to the relatively low value of $\mu_{\rm res}^{-1}$. In fact, substitution of Eq. (6) into Eq. (2) reveals that the weak-localization term can never be significantly larger than $k_B T$.

In summary, we have modeled 2D electron-gas atom scattering data by introducing a helium density-dependent mobility edge which consists of a field-independent trapping term and a weak-localization term. Our estimate of the strength and density dependence of the trapping term agrees well with the data and is consistent with analogous 3D corrections to Boltzmann transport. The temperature dependence of the residual mobility is measured and gives an electron diffusivity of $D \simeq \frac{1}{3}\hbar/m_{\rm el}$ near the mobility edge.

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[1]P. W. Adams and M. A. Paalanen, Phys. Rev. Lett. **58**, 2106 (1987).

[2]P. W. Adams and M. A. Paalanen, Phys. Rev. Lett. **61**, 451 (1988).

[3]A. M. L. Janssen, R. W. van der Heijden, A. T. A. M. de Waele, H. M. Gijsman, and F. M. Peeters, Surf. Sci. **229**, 365 (1990).

[4]M. J. Stephen, Phys. Rev. B **36**, 5663 (1987).

[5]V. V. Afonin, Yu. M. Gal'perin, and V. L. Gurevich, Phys. Rev. B **33**, 8841 (1986).

[6]P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[7]P. W. Adams and M. A. Paalanen, Surf. Sci. **196**, 150 (1988).

[8]P. W. Adams and M. A. Paalanen, Phys. Rev. B **39**, 4733 (1989).

[9]T. R. Kirkpatrick and D. Belitz, Phys. Rev. B **34**, 2168 (1986).

[10]G. Bergmann, Phys. Rev. B **28**, 2914 (1983).

[11]D. Belitz (private communication).

[12]K. W. Schwarz, Phys. Rev. B **21**, 5125 (1980).

[13]M. Saitoh, J. Phys. Soc. Jpn. **42**, 201 (1977).

[14]One expects both extended and localized states to exist at high fields. For instance, both are needed to explain the quantum Hall effect; see H. P. Wei, D. C. Tsui, M. A. Paalanen, and A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1294 (1988).

[15]L. Levine and T. M. Sanders, Phys. Rev. **154**, 138 (1967).

[16]A discrepancy still exists between $\mu_{\rm res}$ on disordered bare H_2 and that of helium-gas atom scattering. The origin of this, however, may lie in inhomogeneous disorder in the bare H_2 studies.

[17]D. C. Glattli, E. Y. Andrei, and F. I. B. Williams, Phys. Rev. Lett. **60**, 420 (1988).

[18]M. A. Paalanen and Y. Iye, Phys. Rev. Lett. **55**, 1761 (1985).